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Further Observations of the Turbulent Fluctuations in a Tidal Current

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FURTHER OBSERVATIONS OF THE TURBULENT FLUCTUATIONS IN A TIDAL CURRENT

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The component of turbulent velocity in the direction of the mean flow has been studied for the tidal current in the Mersey estuary. Two Doodson current meters were used, recording simultaneously on the same photographic paper. The more interesting results were obtained within about 2 m of the bottom, the two meters being supported in a stand, with various vertical and horizontal separations. The periods of the turbulent fluctuations recorded varied from a few seconds up to several minutes. Various methods of analysis have failed to show any predominant periods or bands of periods (when the effects of surface waves have been excluded), and it appears that, as in other types of turbulence, a continuous spectrum of fluctuations is present. Distance-correlation coefficients in the vertical and lateral directions have been computed from the simultaneous recordings, as well as auto-correlation curves from the recordings of the individual meters. Inferring the distance-correlation in the direction of flow from the auto-correlations, the integral scale of the turbulence in this direction is estimated to be of the order of 7 m, compared with 14 m, the mean depth of water. From the simultaneous correlations, it is suggested, tentatively, that the scales in the vertical and lateral directions are of the same order of magnitude and of the order of one-third of the scale in the direction of the mean flow.

1. Introduction

Observations had previously been made (Bowden & Proudman 1949) of turbulent fluctuations in the speed of the tidal current in the River Mersey at various depths, and particularly within about 2 m of the bottom. The instrument used was the Doodson current meter, responding to fluctuations of all periods greater than about 1 s, and, for the bottom observations, it was supported in an iron stand.

In the series of observations with which this paper deals, simultaneous recordings were obtained from two Doodson current meters separated (*a*) in a vertical line, and (*b*) in a horizontal line, transverse to the mean current. It was thus possible to derive, for the first time under these conditions, some values for the distance-correlations, in the vertical and lateral directions, of the longitudinal component of the turbulent velocity. These may be compared with the auto-correlations of the recordings of the individual meters which, on certain assumptions, may be related to the longitudinal distance-correlation. The more interesting results were obtained near the bottom, with the meters supported in the same stand, but observations were also made at other depths.

2. Experimental details

The observations were made between 11 and 26 July 1949 in the River Mersey at Bromborough Buoy (1·37 km, true bearing S 54° W from Dingle Point). The river is 1·54 m wide there at L.W.S. and 2·06 km at H.W.S. (high-water spring tides). The depth of water at the buoy is about 10 m at L.W.S. and 19 m at H.W.S. These figures also represent the maximum depth of water in the section. The bottom is fairly level and hard, consisting mainly of stones, with some shell, in the vicinity of the station. The boat used was the M.V. *William Herdman*, owned by the University of Liverpool and managed by Dr R. J. Daniel.

The current meters were two new models, constructed at the Liverpool Observatory and Tidal Institute under the direction of Dr A. T. Doodson. Certain improvements in design were introduced, but the main features were similar to those of the original model, of which a description has been published (Doodson 1940). A new recorder enabled the indications of the two meters to be recorded side by side on photographic paper, 7 cm wide, driven by a constant-speed electric motor at 6 cm/min. The pressure wheel of the meter has a natural period of about 0·4 s, while the recording galvanometer has a period of 0·9 s and is nearly critically damped, so that a fairly uniform response to fluctuations of periods greater than 2 s may be expected. The two meters were calibrated for speed in the water channel of the Fluid Mechanics Laboratory of the University of Liverpool, the speed of flow being measured by pitot tube.

For the measurements near the bottom, the two current meters were suspended inside the stand previously described (Bowden & Proudman 1949). To separate them vertically one meter was suspended, through a swivel, from an eye-bolt in the top of the stand and the other suspended below it by two chains of the desired length. For the horizontal separation, the two meters were suspended from different positions on a bar across the top of the stand. The meters were free to rotate about a vertical axis, but they were lashed loosely by ropes to the sides or bottom of the stand as necessary, to prevent them from swinging and striking one another or the sides of the stand while it was being lowered or raised. A single electric cable, with seven cores, passed from the recording gear in the boat down to a junction box on the swivel of one of the meters, at which the leads to the two meters were separated.

The arrangement for making measurements with the meters one below the other at other depths was straightforward. The upper meter was suspended by a steel wire in the usual way and the lower meter hung below it by two chains of appropriate length.

For separating the meters horizontally at depths above the bottom, a bar was constructed of angle iron, with a vane at each end to keep it transverse to the current, and holes at intervals along it, from which the meters could be suspended with various separations. The bar itself was suspended by a bridle and a wire leading over a boom at the stern of the boat. When lowered just below the surface, the bar could be seen to orientate itself across the current fairly well, but when lowered to, say, 8 m, the wire led out at a considerable angle to the vertical and sometimes swayed from side to side. In an attempt to locate the bar more accurately, two guy-ropes attached to the ends of the bar, were paid out to equal lengths and secured on either side of the boat. However, for some of the records it was still doubtful whether the bar was transverse to the current and whether some of the fluctuations recorded were not due to oscillations of the bar, so that little weight has been given to the records obtained with this arrangement.

It was intended originally to anchor the boat from both bow and stern, to avoid the necessity of raising the meters for every turn of the tide and so continue observations through slack water. The combined effect of tide and wind on the boat, however, made this impracticable and the previous year's practice was repeated. The meters were raised before the boat began to swing at slack water and were re-lowered when the turn was completed. It seems unlikely that information of much interest was lost by this procedure, as the turbulence becomes very small at low values of the mean current.

3. Analysis of records

3·1. *Summary of records*

Most of the records were of 10 min duration and a total of 114 records, satisfactory from the instrumental point of view, was obtained. Of these, thirty-six were with the meters in the stand, covering two different vertical separations of the meters, 85 and 110 cm, and one horizontal separation, 75 cm. Fifty-one were with the meters suspended freely, one below the other, with separations of 55, 85, 170 and 300 cm. The depth of the upper meter below

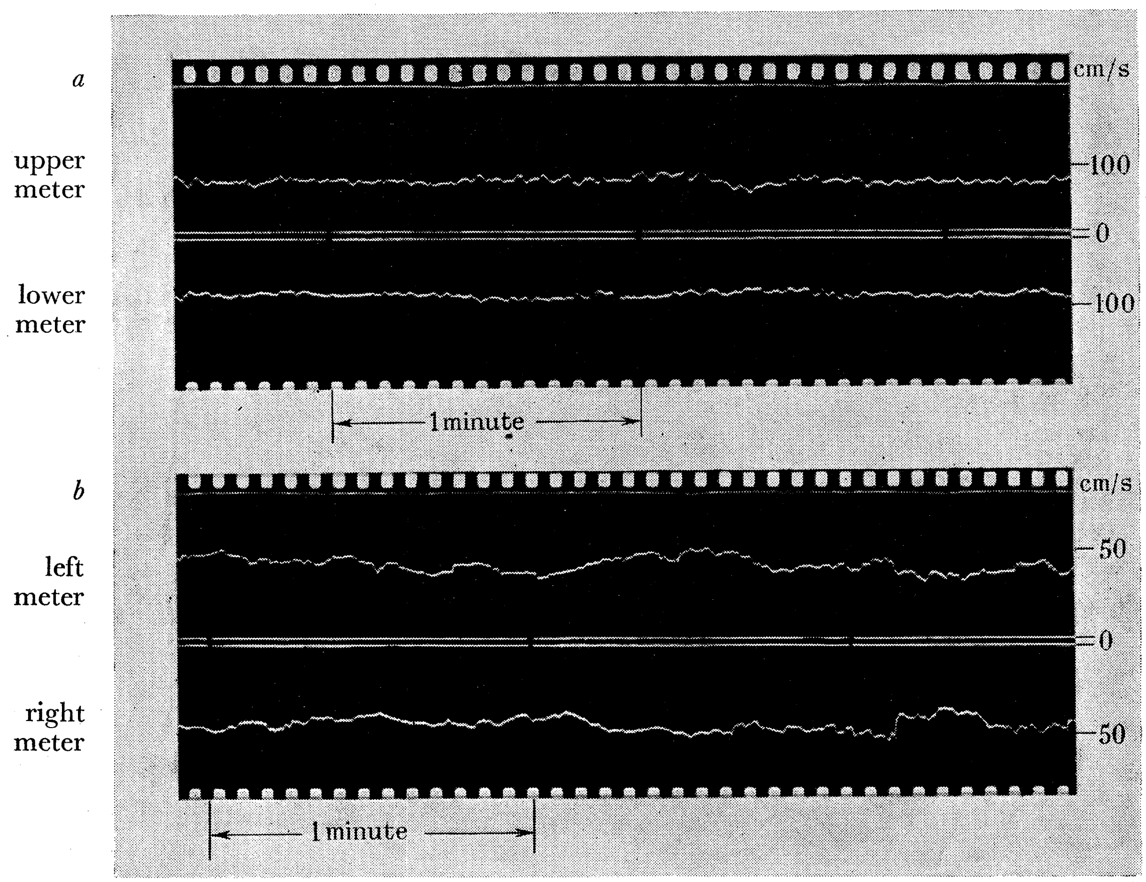

Figure 1. Two examples of simultaneous recordings by two current meters. (*a*) part of record 32*b*, meters 7·3 and 9·0 m below the surface; (*b*) part of record 43*c*, meters separated horizontally by 75 cm, height 1·2 m above the bottom.

the surface varied from 2 to 9 m, approximately. With the meters suspended from the horizontal bar, twenty-seven records were obtained with separations of 90, 150 and 270 cm approximately. The depth of the bar below the surface was nominally 7·3 or 9·1 m, but was probably less, as the bar was carried astern by the current and it was difficult to estimate its depth accurately. Typical examples of the records are shown in figure 1.

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3·2. *General scheme of analysis*

It was at once apparent that the fluctuations on any record contained components of a wide range of periods. In the analysis of previous observations (Bowden & Proudman 1949), a method of graphical smoothing was used to separate fluctuations with periods of the order of a few seconds from those with periods greater than about 30 s. While this procedure appeared justified in an exploratory survey of the fluctuations, it has not been repeated with the present series of records. There is, in fact, no advantage in attempting such a separation unless (*a*) there is internal evidence that the spectrum of the fluctuations contains two or more distinct bands of periods, or (*b*) there are believed to exist two or more different physical processes which are likely to be associated with current fluctuations in different frequency bands. Both (*a*) and (*b*) have been reconsidered in the course of the analysis, but no convincing evidence has been forthcoming under either heading. On the other hand, methods of smoothing have the disadvantage that they may introduce a spurious periodicity into the fluctuations.

In the present analysis, the mean values of the turbulent velocities and the auto-correlation coefficients have been determined without any initial separation of fluctuations of different periods. A system of numerical smoothing was introduced at a later stage in the examination of distance-correlations.

The presence of fluctuations of current due to surface waves was apparent on most records obtained near the surface or at mid-depths. As shown previously, such wave effects are easily distinguishable from the turbulent fluctuations which predominate near the bottom. Most of the detailed analysis has been concentrated on the records taken near the bottom, using the stand, since the wave effects are then much reduced and also movements of the meter relative to the bottom are eliminated.

3·3. *Mean amplitude of fluctuations and the mean current*

Most of the records covered intervals of approximately 10 min, with the exception of numbers 25 *b* and 44 *b* (20 min), 5 and 46 *f* (15 min), 19 (5 min), 21, 22 *a* and 43 *f* (4 min). Each record was divided into sections of 1 min duration and estimates made of the mean currents and mean amplitudes of fluctuation for both meters. Considering the whole record the mean currents U_1 and U_2 (upper and lower meters respectively) and the mean amplitudes of fluctuation u_1 and u_2 (upper and lower meters) were obtained by taking the average over all intervals. The mean current ratio U_1/U_2, the fluctuation amplitude ratio u_1/u_2 and the ratios u_1/U_1 and u_2/U_2 were now deduced. Having grouped them according to the conditions under which the records were made the mean values of U_1, U_2, u_1, u_2, U_1/U_2, u_1/u_2, u_1/U_1, u_2/U_2 for each group were calculated and are shown in table 1.

A feature of the table is that near the surface the values of u_1/U_1 and u_2/U_2 are considerably less than those near the bottom. Where high values do occur near the surface in records 34 *a* to *e* and at mid-depths in the series 31, 32 and 33, there was extensive wave motion at the sea surface. Despite the effect of these waves, however, the values are less than those for the series 41, 42, 43 and 44, where the meters were near the bottom and the influence of surface waves much smaller.

Several groups of records were made with the two meters suspended at the end of a horizontal beam, but only one series, 46 *a* to *f*, has been included in the table. In the others

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TABLE 1

U_1 and U_2 are the mean currents recorded by the upper and lower meters during intervals of 10 min.
u_1 and u_2 are the mean amplitudes of current fluctuation of the meters during the same intervals.
$[U_1]$, $[u_1]$, $[u_1/U_1]$, etc., denote means of U_1, u_1, u_1/U_1, etc., for several 10 min intervals.

1	2	3	upper meter				lower meter				12	13	14	15
	vertical separation of meters	horizontal separation of meters	4 depth of meter	5 distance from bottom	6 $[U_1]$	7 $[u_1]$	8 depth of meter	9 distance from bottom	10 $[U_2]$	11 $[u_2]$	$[U_1/U_2]$	$[u_1/u_2]$	$[u_1/U_1]$	$[u_2/U_2]$
record number	(m)	(m)	(m)	(m)	(cm/s)	(cm/s)	(m)	(m)	(cm/s)	(cm/s)				
10 *a*, *b*	0·8	—	3·7	meters suspended vertically from the surface	118	2·4	4·5	meters suspended vertically from the surface	127·5	3·1	0·93	0·84	0·021	0·024
11, 12 *a*, *b*, *c*	—	—	3·7		107	4·0	—		—	—	—	—	0·037	—
28 *a*, *b*	1·7	—	3·7		71·5	2·4	5·4		66	3·4	1·09	0·73	0·033	0·051
34 *a*, *b*, *c*, *d*, *e*	3·0	—	5·5		81·5	4·3	8·5		83	4·7	0·97	0·89	0·052	0·057
5	0·6	—	7·3		85·5	2·5	7·9		93	2·3	0·92	1·10	0·030	0·024
18 *a*, *b*, 19, 20 *a*, *b*, 21, 22 *a*, *b*, *c* 23, 24, 25 *b*	0·9	—	7·3		120	4·4	8·2		118	4·0	1·03	1·05	0·037	0·047
31 *a*, *b*, *c*, 32 *a*, *b*, *c*, 33 *a*, *b*, *c*	1·7	—	7·3		68	4·3	9·0		69·5	4·8	1·00	0·89	0·062	0·070
8 *a*, *b*	0·6	—	9·1		157	6·2	9·7		159	6·0	0·99	1·05	0·040	0·037
15 *a*, *b*, 16 *a*, *b*	0·9	—	9·1		77	< 1·0	10·0		75·5	< 1·0	0·98	—	< 0·015	< 0·015
29 *a*, *b*	1·7	—	9·1		62	4·0	10·8		61	5·2	1·01	0·78	0·065	0·085
41 *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*	0·8	—	meters in stand on bottom	1·8	50	5·5	meters in stand on bottom	1·0	42·5	6·2	1·11	0·99	0·112	0·146
42 *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*	1·1	—		2·1	43	3·6		1·0	38	3·8	1·14	0·95	0·083	0·101
43 *a*, *b*, *c*, *d*, *e*, *f*, 44 *a*, *b*, *c*, 45 *c*, *d*	—	0·8		1·2	47	5·3		1·2	47	6·0	1·0	0·90	0·115	0·129
46 *a*, *b*, *d*, *e*, *f*	—	1·5	9·1	meters on horizontal beam	55	1·9	9·1	meters on horizontal beam	60	2·3	0·92	0·84	0·35	0·038

there was reason to suspect that some of the fluctuations were due to oscillation of the bar.

3·4. *Dependence of the fluctuation ratio u/U on the distance from the bottom*

This investigation was restricted to those records made when the meters were fixed in the stand on the river bottom. The value of u/U (i.e. u_1/U_1 or u_2/U_2) was plotted against height above the bottom for each record (figure 2). Mean values of u/U were found, and their graph shows that the ratio appears to decrease as the distance from the bottom

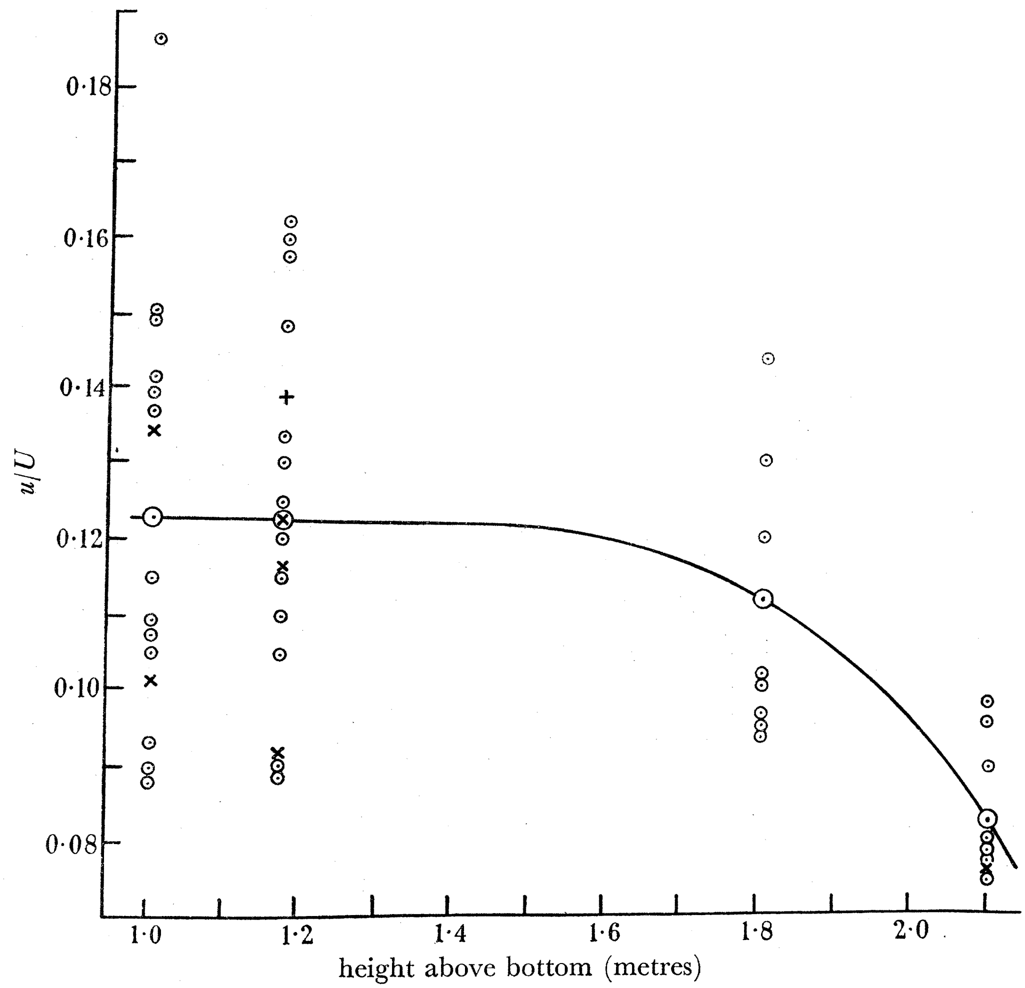

FIGURE 2. Variation of current fluctuation ratio u/U with height above bottom. ⊙ denotes a single value, × and + denote two and three identical values, respectively.

TABLE 2. VARIATION OF STANDARD DEVIATION OF u/U WITH HEIGHT ABOVE BOTTOM

standard deviation of u/U	0·027	0·021	0·017	0·009
height above bottom (m)	1·0	1·2	1·8	2·1

increases. The distribution of values of u/U about their means shows variation, and it was found that the standard deviation diminishes as the height above the bottom increases (table 2). Owing to the restricted number of observations, particularly at distances of 1·8 and 2·1 m from the bottom, no great significance can be attached to this result at present.

3·5. *Distance correlation and auto-correlation*

Sixteen records were selected for detailed analysis, consisting of one made near the surface, three midway between surface and bottom, and twelve near the river bottom. By means of a suitably calibrated scale the magnitude of the current on each trace was measured at the same instant of time, and readings were repeated at regular intervals of approximately 1·25 s over the whole record. It was now possible to calculate the distance correlation coefficient r defined by the relation

$$r = \frac{\sum_{t=0}^{T} (u_1)_t (u_2)_t}{\left\{\sum_{t=0}^{T} (u_1)_t^2 \sum_{t=0}^{T} (u_2)_t^2\right\}^{\frac{1}{2}}}.$$

The total time occupied by the record is denoted by T, which is in units of 1·25 s. $(u_1)_t$ and $(u_2)_t$ represent the deviations of the two current amplitudes from their respective means U_1 and U_2 at time t, U_1 and U_2 being averages over time T.

For several of the pairs of series auto-correlation coefficients were found. The auto-correlation for time interval τ, denoted by r_τ, is given by

$$r_\tau = \frac{\sum_{t=0}^{T-\tau} (u_1)_t (u_1)_{t+\tau}}{\left\{\sum_{t=0}^{T-\tau} (u_1)_t^2 \sum_{t=0}^{T-\tau} (u_1)_{t+\tau}^2\right\}^{\frac{1}{2}}}.$$

The records analyzed in this way extended over periods of approximately 10 min, and thus each series of readings contained about 500 elements. This length of record was chosen because it was known from previous experience that auto-correlation coefficients calculated from records of 2 or 3 min duration showed considerable variation when successive lengths of the same record were used. It was found that similar variation occurred with the values of distance correlation coefficients. Three groups of 201 readings, covering a time interval of 250 s, were taken from each of five records, and the values of r are shown in table 3 (*a*).

TABLE 3

(*a*)

record number	distance correlation coefficient r: readings 0–200	readings 100–300	readings 200–400
10 *b*	0·53	0·69	0·77
33 *a*	0·61	0·57	0·49
41 *a*	0·35	0·56	0·63
42 *c*	0·46	0·46	0·51
44 *b*	0·36	0·50	0·71

(*b*)

record number 44 *b* readings	r
0–500	0·63
100–600	0·66
200–700	0·65
300–800	0·66
400–900	0·68

Using one of the longest records available r was found for several groups of 501 readings, extending over 625 s. The values obtained are given in table 3 (*b*) and show a smaller spread than those in (*a*).

Table 4 contains all the values of the distance correlation coefficient and auto-correlation coefficient and details of the situation of the two current meters in each case.

To illustrate further the variation of the auto-correlation coefficient r_τ a representative group of correlograms is shown in figure 3. The curves for record no. 45 *c* are typical of those

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Table 4. Correlations

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
record number	vertical separation of meters (cm)	horizontal separation of meters (cm)	depth of meter (cm)	distance of meter from bottom (cm)	distance correlation coefficient r	auto-correlation coefficients r_τ time intervals in seconds										time τ_1 when $r_\tau = r$ (s)	mean current U (cm/s)	$U\tau_1$ (cm)
						1·25	2·50	3·75	5·00	6·25	7·50	8·75	10·00	11·25	12·50			
10 *b*	85	—	370	—	0·64	0·60	0·53	0·52	0·40	0·34	0·31	0·20	0·17	0·07	0·03	1·25	107·4	130
			455	—		0·65	0·54	0·52	0·38	0·28	0·25	0·12	0·08	0·08	0·07			
20 *b*	85	—	730	—	0·78	0·75	0·66	0·65	0·60	0·54	0·53	0·50	0·49	0·48	0·45	0·95	106·6	100
			815	—		0·73	0·69	0·69	0·65	0·62	0·62	0·60	0·58	0·56	0·52			
32 *a*	170	—	730	—	0·44	0·42	0·40	0·36	0·18	0·11	0·14	0·06	0·04	0·06	0·04	2·45	78·9	190
			900	—		0·55	0·48	0·46	0·30	0·27	0·22	0·16	0·13	0·10	0·07			
33 *a*	170	—	730	—	0·55	0·46	0·40	0·66	0·35	0·21	0·41	0·29	0·08	0·08	0·17	—	—	—
			900	—		0·65	0·60	0·70	0·52	0·41	0·50	0·40	0·28	0·28	0·25			
41 *a*	85	—	—	180	0·57	0·75	0·69	0·56	0·50	0·45	0·42	0·40	0·37	0·37	0·36	3·75	40·5	150
			—	95		0·72	0·65	0·57	0·51	0·48	0·40	0·39	0·38	0·35	0·32			
41 *f*	85	—	—	180	0·49	0·85	0·73	0·66	0·61	0·56	0·54	0·51	0·50	0·49	0·47	6·70	51·4	340
			—	95		0·80	0·65	0·52	0·50	0·43	0·41	0·39	0·35	0·32	0·26			
42 *a*	110	—	—	210	0·59	0·83	0·78	0·74	0·67	0·63	0·57	0·53	0·52	0·48	0·42	6·90	43·7	300
			—	100		0·93	0·85	0·77	0·68	0·60	0·57	0·51	0·48	0·45	0·43			
42 *b*	110	—	—	210	0·29	—	—	—	—	—	—	—	—	—	—	—	—	—
			—	100														
42 *c*	110	—	—	210	0·53	0·90	0·79	0·68	0·58	0·45	0·35	0·25	0·17	0·12	0·05	8·75	40·5	350
			—	100		0·92	0·84	0·77	0·72	0·68	0·65	0·63	0·58	0·55	0·49			
42 *d*	110	—	—	210	0·39	—	—	—	—	—	—	—	—	—	—	—	—	—
			—	100														
43 *a*	—	75	—	120	0·62	0·90	0·81	0·74	0·68	0·62	0·60	0·57	0·51	0·48	0·45	7·00	41·5	290
			—			0·90	0·83	0·76	0·71	0·66	0·62	0·59	0·56	0·55	0·54			
43 *c*	—	75	—	120	0·73	—	—	—	—	—	—	—	—	—	—	—	—	—
43 *e*	—	75	—	120	0·74	—	—	—	—	—	—	—	—	—	—	—	—	—
44 *a*	—	75	—	120	0·61	—	—	—	—	—	—	—	—	—	—	—	—	—
44 *b*	—	75	—	120	0·63	—	—	—	—	—	—	—	—	—	—	—	—	—
45 *c*	—	75	—	120	0·55	0·86	0·75	0·66	0·59	0·53	0·48	0·45	0·40	0·37	0·36	6·25	46·8	290
						0·86	0·76	0·67	0·61	0·57	0·53	0·47	0·42	0·39	0·38			

for records made just above the river bottom. In general, such curves are similar in shape, showing a steady decrease in r_τ with increasing values of τ. Usually the pair of curves corresponding to simultaneous traces from the two meters lie fairly close together. An exception is record 42*c* which, as seen from the set of coefficients in table 4, would give a pair of curves with marked differences in their gradients. Considering the curves from different records there are appreciable differences in the rate at which they approach the axis of τ. The curves for records 33*a* and 10*b* are good examples of those at mid-depths and near the surface respectively. Each curve shows that r_τ undergoes oscillations of decreasing amplitude superimposed on a steady decay as τ increases. Where these oscillations are found surface waves of similar period were noted during the production of the records, thus confirming the results of the work of 1949 which attributed such oscillations to wave currents.

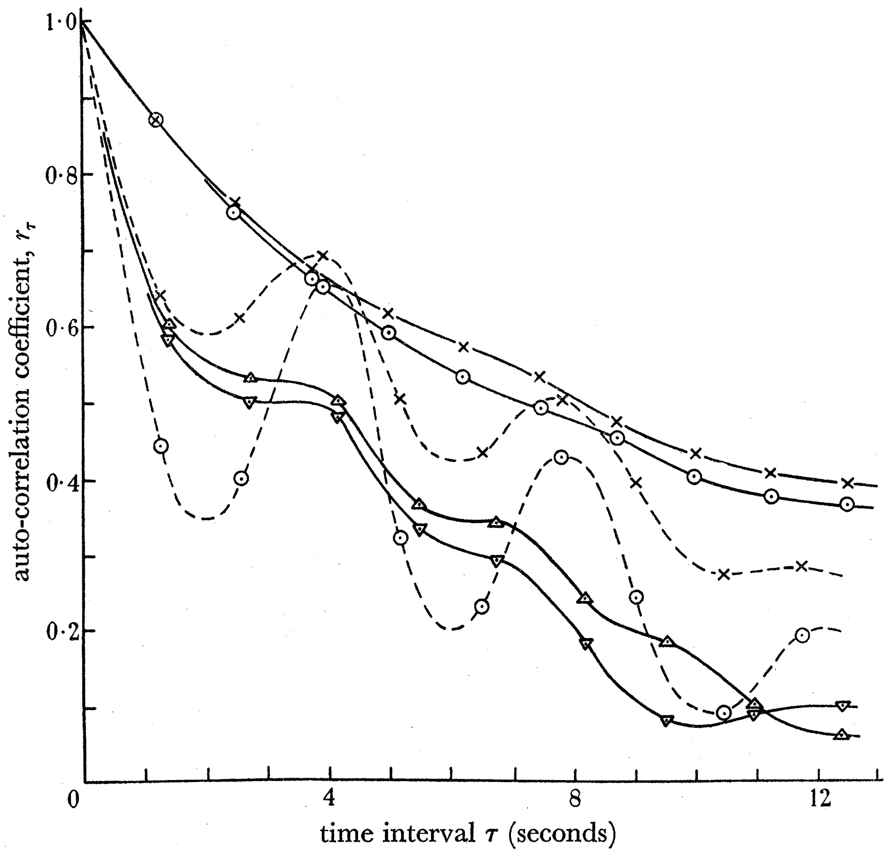

FIGURE 3. Auto-correlation curves at various depths. Record 10*b*: —△— depth 3·7 m, —▽— depth 4·55 m. Record 33*a*: -- ⊙-- depth 7·3 m, -- × -- depth 9·0 m. Record 45*c*: — ⊙—, — × —, height above bottom 1·2 m (meters side by side).

The values of the distance-correlation coefficient r cover a wide range, even for records where the meters had the same spacing and position relative to the surface or bottom of the river. From table 4 it can be seen that the largest group of records made under similar conditions is that where the meters had a horizontal separation of 75 cm and were about 120 cm from the river bottom. Here the values of r are consistently high, the average being 0·65 and the standard deviation 0·07. The group of four records where the meters had a vertical separation of 110 cm, with the lower 100 cm from the bottom, shows more divergent values of r, the average being about 0·45 and the standard deviation 0·13. Near the surface there are a number of fairly high values of r, particularly that of 0·78 for record no. 20 *b*.

3·6. *Spectrum of the fluctuations*

Let a spectrum function $W(f)$ be defined such that fluctuations having frequencies between f and $f+\mathrm{d}f$ contribute $[u^2]\, W(f)\, \mathrm{d}f$ to the total value of $[u^2]$. Then $W(f)$ is related to r_τ by the transformation

$$W(f) = 4\int_0^\infty r_\tau \cos 2\pi f\tau \,\mathrm{d}\tau.$$

(See, for example, Taylor 1938).

To apply this equation it is necessary that the correlogram should be extended until r_τ approaches the axis of τ. This has been done for two records, 41 *a* and 43 *a*, as shown in figure 4, taking the mean value of r_τ for the two traces in each case. The points do not approach the axis asymptotically but appear to oscillate about it, and these oscillations have been smoothed out in drawing the curves. This is considered a justifiable, and in fact a necessary, procedure, since the values of τ are of the order of 1 min and thus an appreciable fraction of the 10 min length of record. The standard deviation of the errors in r_τ is estimated below to be of the order of 0·15, and the square root of the covariance of near values r_τ and r_{τ_1} of the same order, so that the oscillations appearing are probably spurious.

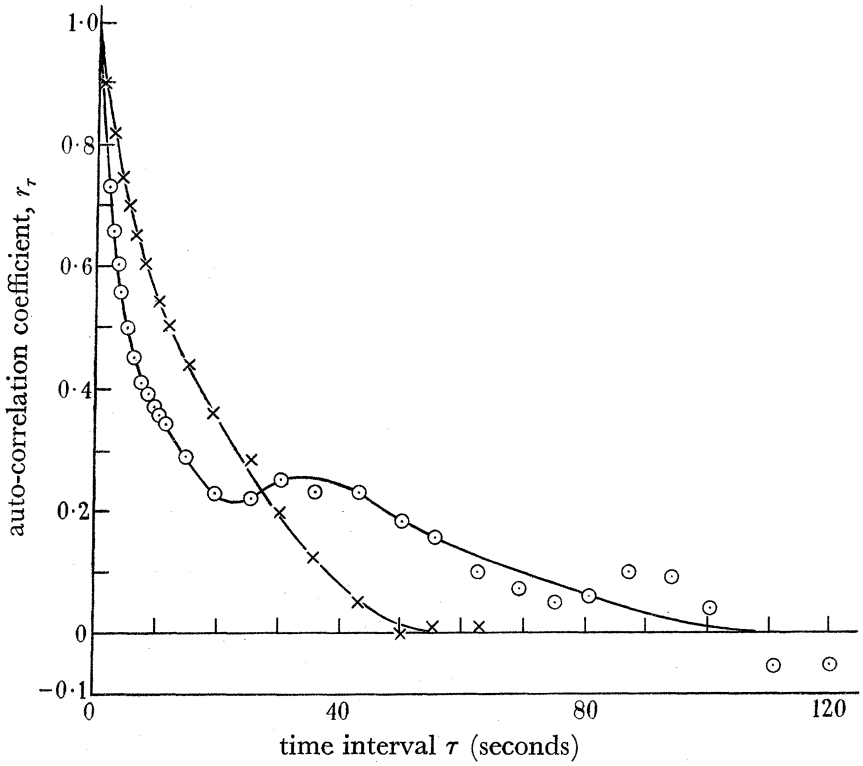

FIGURE 4. Auto-correlation curves for longer time intervals. —⊙— record 41 *a*, —×— record 43 *a*. Each set of points represents the mean from the two meters.

From the curves of r_τ, $W(f)$ has been computed numerically for records 41 *a* and 43 *a*, and the resulting curves of $W(f)$ against f are shown in figure 5. Both curves show the largest values of $W(f)$ for the lowest values of f. Apart from a small peak at $f = 0·067\,\mathrm{s}^{-1}$

on the curve for 41 *a*, $W(f)$ decreases continuously with increasing f. These general features are in agreement with the results for the spectrum of turbulence in pipes and wind-tunnels. There is no significant evidence of any particular band of frequencies standing out from the continuous spectrum.

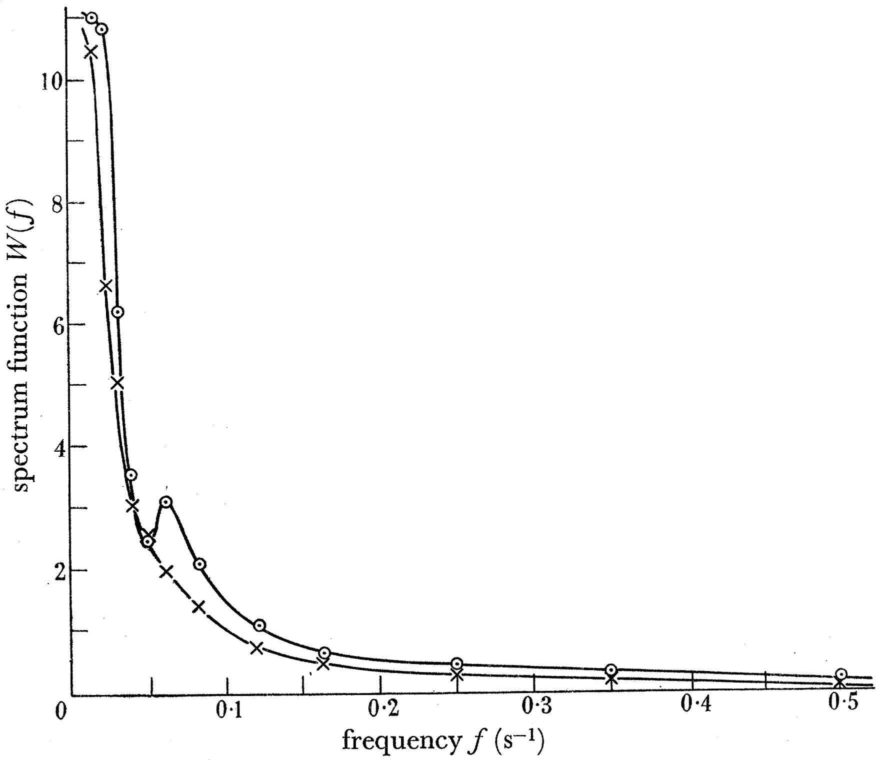

FIGURE 5. Spectrum function curves. —⊙— record 41 *a*, —×— record 43 *a*.

The variance of computed values of r_τ, when τ is sufficiently large for the true value of r_τ to have become very small, has been shown by Bartlett (1946) to be given approximately by

$$\operatorname{var}(r_\tau) = \frac{1}{T}\int_{-\infty}^{\infty} r_\tau^2\, d\tau,$$

where T is the total length of record. Similarly, the covariance of values r_τ and $r_{\tau 1}$ is then given by

$$\operatorname{cov}(r_\tau, r_{\tau 1}) = \frac{1}{T}\int_{-\infty}^{\infty} r_\tau r_{\tau 1}\, d\tau.$$

The above equation gives the standard deviation of r_τ when τ is large as 0·15 for record 41 *a* and 0·17 for 43 *a*. This comparatively large uncertainty in the shape of the 'tail' of the correlogram should be borne in mind in considering figures 4 and 5. Since every point on the spectrum curve involves an integration over the whole correlation curve, there appears to be a limit to the usefulness of the spectrum function as a method of representing the results of the analysis of a restricted length of record.

3·7. *Separation of periods by numerical smoothing*

The distance correlation coefficients shown in table 4 (column 6) are due to fluctuations with a wide range of periods so an attempt has been made to reduce this range in stages and interpret the resulting changes of the coefficients. In the earlier paper (Bowden & Proudman 1949) long-period fluctuations were extracted from the current records by making a tracing of each in which all short period variations were smoothed out. Subtraction of the current amplitude of the tracing from that of the original record gave a residual series of fluctuations of short period. The current was then measured at equal intervals of time throughout both sections and correlogram analysis carried out in each case. Owing to the complex and variable nature of the records this type of separation was inevitably rather arbitrary, and its effects were likely to alter from one record to another. It was thus desirable to substitute a method which could be applied with uniform efficiency to all records, especially since the present investigation involved the calculation of distance correlation coefficients requiring the use of pairs of traces simultaneously. Numerical smoothing appeared to satisfy this condition, and possessed the additional advantage that separation could be easily effected at different places in the range of periods under consideration. Knowing that the current fluctuations probably contained random elements it was decided to adopt smoothing employing a simple moving average. This type of average would have least effect on any random components in the records and thus reduce the possibility of producing spurious oscillations in the smoothed series (Kendall 1946).

The process was applied to several 10 min lengths of record where current readings for both meters had been made at regular intervals of approximately 1·25 s. Beginning at the first reading in any series and taking the first n, where n is odd, the mean value was calculated, giving the value of the moving average for the central reading in the group. Beginning at the second reading and selecting the next n a further mean was found, and this operation was repeated over the whole series. The effect of subtracting the moving averages from the appropriate readings was to produce a new series in which fluctuations having periods greater than $1{\cdot}25\ (n-1)$ s were considerably attenuated. The initial use of high values of n removed fluctuations of long period, and the systematic use of decreasing values reduced the remaining frequency range. For every pair of series a number of values of n between 121 and 3 were used, giving 'periods of smoothing' in the range 150 to 2·5 s. The upper limit to the value of n was set by the length of the smoothed series at this stage. Application of a moving average containing n readings entails a loss of $(n-1)$ readings when the smoothed series is obtained. Since large variations were likely to occur in correlation coefficients calculated with short sections of the same trace the greatest value of n was limited to 121, making available a smoothed series of 7·5 min duration. With values less than 121 the smoothed series were longer, but in finding the distance-correlation coefficient r only the central 7·5 min common to all was used. Taking the entire length of smoothed series in each case gave differences in the values of r as great as 0·4. The average difference was much smaller, however, being less than 0·02. In some instances where small values of n were used it was apparent that the values of r were governed by a few exceptionally large or small current readings, this being very obvious in record no. 42 *a*. Because of these variations in the value of r the significance of values below 0·15 seems to be small. The significance of the values of auto-correlation coefficients is examined in § 3·6, and the results obtained indicate that values of r_τ below 0·15 have little significance. This evidence supports the selection of a similar value for r.

In addition to the correlation coefficient the mean-square fluctuation velocity over each pair of traces was found. Denoting this mean-square velocity by $[u^2]$, the total number of readings in each series by N and the time occupied by the record by T

$$[u^2] = \frac{1}{2N}\left\{\sum_{t=0}^{T}(u_1)_t^2 + \sum_{t=0}^{T}(u_2)_t^2\right\},$$

where $(u_1)_t$ and $(u_2)_t$ are the velocities defined in § 3·5.

Values of r were plotted against the period of smoothing P and the graphs are shown in figure 6. The outstanding feature of the curves is that in general there is a continuous decrease in the value of the correlation coefficient as the fluctuations of longer period are removed by decreasing the period of smoothing. Apart from regions where the values of r have doubtful significance the only deviation from this general behaviour occurs in the curve for record no. 45 *c*, where r reaches a minimum for a value of P in the region of 50 s and a maximum when P is further reduced to approximately 30 s. The curves do, however, exhibit differences in shape, even with records 43 *a* and 45 *c* where the position of the two meters was similar. This may be due to differences in the spectrum of fluctuations.

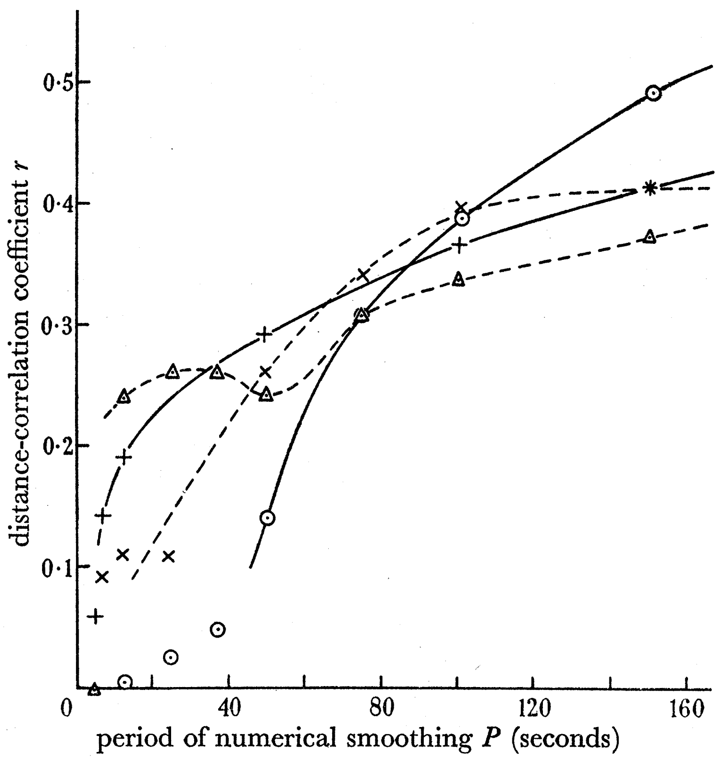

FIGURE 6. Variation of distance-correlation coefficient with period of numerical smoothing. —+— record 41 *a*, – – × – – record 42 *a*, — ⊙— record 43 *a*, – – △– – record 45 *c*.

With smoothing over a period as great as 150 s the values of r are still appreciably less than those before smoothing. There is an average decrease of 27 % in r for this smoothing, and thus fluctuations with periods greater than 150 s seem to be of some importance. As P is reduced below 150 s r decreases more rapidly and the gradients of the curves have high values where P lies between 25 and 100 s. For records 43 *a* and 42 *a*, however, the significance of r becomes doubtful where P is reduced to values of 55 and 30 s respectively.

It would therefore appear that fluctuations of the order of 60 to 100 s have high correlation, while those of shorter period have correlations which vary in magnitude from one record to another.

The relations between $[u^2]$ and P shown in figure 7 follow a pattern similar to those between r and P, the greatest changes in r and $[u^2]$ occupy approximately the same range of values of P. A feature common to all four records is that $[u^2]$ is still greater than 3·0 $(\text{cm/s})^2$ for values of P less than 10 s.

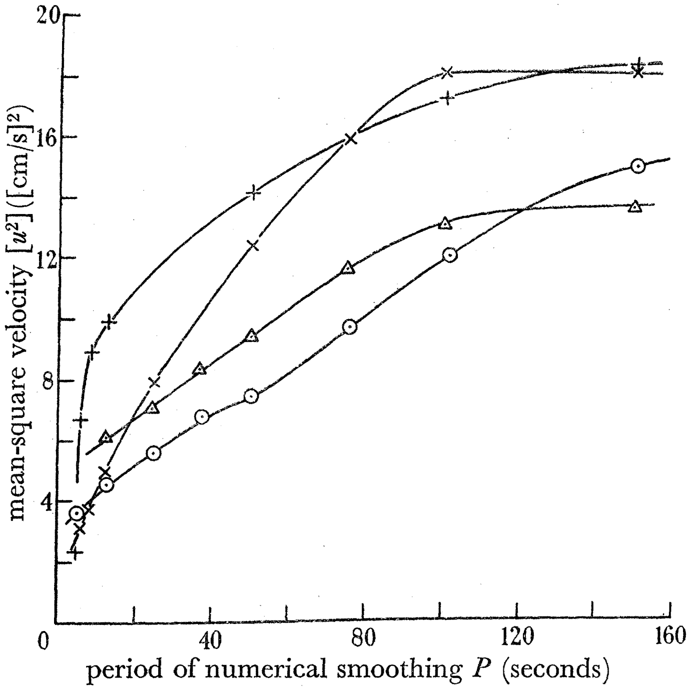

FIGURE 7. Relation between mean square value of turbulent velocity and period of numerical smoothing. —+— record 41*a*, —×— record 42*a*, —⊙— record 43*a*, —△— record 45*c*.

4. Discussion of results

4·1. *Introductory*

The evidence indicating that the shorter-period fluctuations were due to turbulence was summarized in the previous paper (Bowden & Proudman 1949), while judgement was suspended in the case of the long-period fluctuations. An alternative explanation of the latter would be to attribute them to internal waves. This question is discussed more fully in § 4·5 below, without reaching any definite conclusion apart from the negative one that, with the data available, one cannot predict any discrete periods or band of periods likely to be due to internal waves. On the other hand, the evidence discussed in both earlier papers (Bowden 1947; Bowden & Proudman 1949) shows that the long-period fluctuations, like those of short period, have features consistent with their being turbulent in character. In the present investigation, the fluctuations have been analyzed as a single set, but if they fell naturally into two bands of periods, this fact might have been expected to emerge.

It is therefore suggested that the current fluctuations which have been observed, throughout the whole range of periods, should be regarded as similar in character and probably all associated with the turbulent flow of the tidal current. The discussion from § 4·6 onwards is based on this hypothesis.

In §§ 4·2 to 4·4, several possible sources of error in the computation of the distance-correlation coefficients are discussed.

4·2. *Effects of surface waves*

If surface waves of a single period T are present among the fluctuations, it is known that their effect on the auto-correlation curve is to cause an undamped oscillation in r_τ of period T. If the periods of the waves extend over a band, centred at T, the corresponding oscillation in r_τ is damped. This effect has already been mentioned in connexion with particular records when it has appeared and no further discussion is needed.

Considering the effect on the distance-correlation in the vertical direction, it is seen that surface waves acting alone would give a coefficient of 1, independent of the separation, since the phase of the motion is the same at all points in a vertical line.

To determine the correlation in the horizontal direction transverse to the mean current U, let α be the angle between U and the direction of propagation of the waves. Let l be the horizontal separation of the two meters perpendicular to U. Let u_1'', u_2'' be the components parallel to U of the particle velocities due to the waves at the two meters respectively. Then u_1'' and u_2'' may be represented by

$$u_1'' = u_0 \cos\alpha \cos 2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right),$$

$$u_2'' = u_0 \cos\alpha \cos 2\pi\left(\frac{x-l\sin\alpha}{\lambda}-\frac{t}{T}\right),$$

where λ is the wave-length and u_0 the amplitude of the horizontal component of the particle velocity at the depth considered. Then the correlation coefficient r'', defined by

$$r'' = \frac{[u_1'' u_2'']}{\{[(u_1'')^2]\,[(u_2'')^2]\}^{\frac{1}{2}}},$$

where the square brackets [ ] denote mean values with respect to time, is given by

$$r'' = \cos\left(\frac{2\pi l \sin\alpha}{\lambda}\right).$$

If $\alpha = 0$ or π, i.e. the waves are travelling in the same direction as U, in either sense, $r'' = 1$, independent of the separation l. In other cases, r'' will still be nearly 1 if $l \ll \lambda/2\pi$.

The above equations for u_1'' and u_2'' assume the waves to be long-crested. For short-crested waves, the value of r'' will, in general, be less than that given by the above equation, but the decrease will be small if l is small compared with the crest-length.

To cover the case of wave currents superimposed on turbulent fluctuations, let the resultant fluctuations at the two meters be u_1 and u_2 respectively, where

$$u_1 = u_1' + u_1'', \quad u_2 = u_2' + u_2''.$$

Let
$$r' = \frac{[u_1' u_2']}{\{[(u_1')^2]\,[(u_2')^2]\}^{\frac{1}{2}}}, \quad r'' = \frac{[u_1'' u_2'']}{\{[(u_1'')^2]\,[(u_2'')^2]\}^{\frac{1}{2}}},$$

while
$$[u_1' u_1''] = [u_2' u_2''] = [u_1' u_2''] = [u_1'' u_2'] = 0.$$

If r is defined by
$$r = \frac{[u_1 u_2]}{\{[u_1^2]\,[u_2^2]\}^{\frac{1}{2}}},$$

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it follows that
$$r = \frac{r' + abr''}{\{(1+a^2)(1+b^2)\}^{\frac{1}{2}}},$$
where
$$a^2 = [(u_1'')^2]/[(u_1')^2], \quad b^2 = [(u_2'')^2]/[(u_2')^2].$$

If u_1', u_2' are the turbulent fluctuations and u_1'', u_2'' the wave currents, then $r'' = 1$ for vertical separations and very nearly 1 for the horizontal separations.

If $r'' = 1$,
$$r = (r' + ab)/\{(1+a^2)(1+b^2)\}^{\frac{1}{2}}.$$
If, further, $a = b$,
$$r = (r' + a^2)/(1+a^2).$$

As an example, suppose $r' = 0{\cdot}5$, $a = b = 1$, i.e. the wave currents have the same root-mean-square value as the turbulent fluctuations, then $r = 0{\cdot}75$. Thus, in the presence of waves the distance-correlations may appear considerably larger than the true correlations due to the turbulence alone. The high values of r shown in table 4 for records 10 *b* and 20 *b* are probably to be accounted for in this way. The safest course is to disregard distance-correlation coefficients obtained from records for which the auto-correlation curves show wave motion to have been appreciable.

4·3. *Effect of uncorrelated disturbances at the two meters*

An estimate may be made of the error likely to be introduced if, due to disturbance of the flow by the presence of the meter, or to any other cause, the current recorded at each meter includes uncorrelated disturbances as well as the fluctuations due to the field of turbulence. Let u_1'', u_2'' be the disturbances, then, with the notation of the preceding paragraph, $r'' = 0$ and $r = r'/\{(1+a^2)(1+b^2)\}^{\frac{1}{2}}$.

Hence the apparent correlation coefficient is reduced in a ratio depending only on a and b. If a and b are independent of the separation of the meters, this ratio is also independent of the separation. Suppose $a = b = 0{\cdot}5$, i.e. the disturbances have half the root-mean-square value of the turbulent fluctuations, then $r = 0{\cdot}8r'$. Thus the disturbances would have to reach a considerable fraction of the turbulent fluctuations before the apparent correlation coefficient is seriously reduced.

4·4. *Effect of a common trend in the current at the two meters*

Let the mean current be changing uniformly during the duration of the record. The equations of § 4·2 may be applied if u_1'', u_2'' are taken as
$$u_1'' = \Delta U_1\left(\frac{2t}{T} - 1\right),$$
$$u_2'' = \Delta U_2\left(\frac{2t}{T} - 1\right),$$
where t is the time measured from the beginning of the record and T is the total duration of the record. The mean current increases by $2\Delta U_1$ and $2\Delta U_2$ respectively at the two meters in the time T.

Then $r'' = 1$,
$$[(u_1'')^2] = \tfrac{1}{3}(\Delta U_1)^2, \quad [(u_2'')^2] = \tfrac{1}{3}(\Delta U_2)^2.$$
Thus
$$a^2 = \frac{1}{3}\frac{(\Delta U_1)^2}{[(u_1')^2]}, \quad b^2 = \frac{1}{3}\frac{(\Delta U_2)^2}{[(u_2')^2]},$$
and
$$r = (r' + ab)/\{(1+a^2)(1+b^2)\}^{\frac{1}{2}}.$$

If, for example, $r' = 0{\cdot}5$, $(\Delta U_1)^2 = [(u_1')^2]$, $(\Delta U_2)^2 = [(u_2')^2]$, then $a^2 = b^2 = \frac{1}{3}$ and $r = 0{\cdot}625$. This example represents a comparatively large change in the mean current, which would be apparent from the minute-mean values. On most of the records the trend is considerably less than this and the corresponding error in r is probably negligible. A few records were taken when the mean current was changing rapidly and gave unusually high values of r in accordance with this effect. Such records have not been included in the discussion.

4·5. *Possible effect of internal waves*

It had been suggested previously that, while the shorter-period fluctuations in a tidal current were probably due to turbulence, the longer-period fluctuations might be due to internal waves. To investigate this possibility, observations of temperature and salinity were made at various depths at Bromborough Buoy, at different states of the tide, during July 1949, although not simultaneously with the turbulence observations. Table 5 shows the mean differences in σ_t between surface and mid-depth and between mid-depth and approximately 2 m above the bottom. σ_t is defined by $\rho = (1+\sigma_t/1000)\,\text{gcm}^{-3}$, where ρ is the density.

TABLE 5

	$\Delta\sigma_t$		in layer 2 to 7 m above bottom		
state of tide	surface to mid-depth	mid-depth to bottom	$10^3\,\frac{d\rho}{dz}$ (gcm^{-4})	$\frac{dU}{dz}$ (s^{-1})	R_i
high water	0·32	0·07	0·014	small	large
half ebb	0·09	0·18	0·036	0·08	0·54
low water	0·86	0·31	0·062	small	large
half flood	0·43	0·15	0·030	0·08	0·45

The occurrence and stability of internal waves in a fluid in which the density and mean velocity vary continuously with depth is a problem for which no general solution appears to have been given. For the particular case of water of infinite depth and uniform gradients of density and velocity, Taylor (1931) showed that stable internal waves are possible if

$$R_i > \tfrac{1}{4},$$

where R_i is the Richardson number, defined by

$$R_i = \frac{g}{\rho}\frac{d\rho}{dz}\Big/\left(\frac{dU}{dz}\right)^2.$$

If $R_i < \frac{1}{4}$, no internal waves, stable or unstable, can exist.

In the above case R_i will be least at half tide when dU/dz is greatest. From the analysis of the mean current, there appeared to be little difference in U from the surface to mid-depth (about 7 m above bottom), where $U = 112{\cdot}8$ cm/s for the amplitude of the M_2 constituent. At 2 m above the bottom $U = 71{\cdot}7$ cm/s. Assuming the gradients to be uniform, values of dU/dz, $d\rho/dz$ and R_i have been computed for the lower layer of water, between approximately 2 and 7 m above the bottom, and are given in table 5.

For the upper layer, $d\rho/dz$ is of the same order and dU/dz is less, so that R_i will be somewhat greater. Since $R_i > \frac{1}{4}$, the Taylor criterion would suggest that stable internal waves are possible, at least down to 2 m above the bottom. In the first 2 m, dU/dz is considerably larger, and there are no data on $d\rho/dz$, so that R_i may be less than $\frac{1}{4}$. The physical interpretation of the criterion would then be obscure, however, especially as neither of the conditions postulated, i.e. infinite depth and uniform gradients, are valid near the bottom.

Assuming that internal waves may exist, their period remains indeterminate, since the criterion is independent of period. Standing waves of discrete frequencies seem unlikely in the absence of barriers across the estuary. Progressive waves appear possible, but there are no known agencies present which would favour the generation of waves of one period or group of periods rather than another. The interpretation of the long-period current fluctuations observed would therefore seem to lie between (*a*) turbulence and (*b*) a continuous spectrum of progressive internal waves. It does not seem possible with the present data to distinguish between these two possible causes. In the case of surface waves, their large phase velocity relative to the mean current, and their rapid attenuation with depth lead to methods of separating them readily from turbulence. Internal waves, on the other hand, would have velocities of the same order as the mean current, while their variation of amplitude with depth is not known. It might be possible to devise experimental arrangements which would enable the distinction to be made, e.g. simultaneous observations with two current meters separated longitudinally by a distance comparable with the scale of the turbulence (or the wave-length of the internal waves).

4·6. *Longitudinal distance-correlation inferred from auto-correlation*

It is usually assumed in the study of turbulence that, if the turbulent velocity u is small compared with the mean velocity U, the change in u observed at a fixed point in a short time τ is due chiefly to the convection of the pattern of turbulence past the point by the mean current (Taylor 1938). The change in the turbulent velocity of a particular element of water during the time τ is taken to be negligible. On this assumption the auto-correlation coefficient r_τ corresponding to a time interval τ may be regarded as the longitudinal distance-correlation coefficient for a separation $x = U\tau$.

Figure 8 shows the values of r_τ plotted as a function of $x = U\tau$ for all records near the bottom. The curves show a similar trend and lie within a broad band. If the curves corresponding to records near the surface and at mid-depths are plotted in the same way, then, apart from the oscillations due to surface waves, they also have a similar form. It is not possible, considering the spread of the curves, to discern any significant difference in the shape of the curves or the scale of the turbulence at different levels.

The mean curve for all records near the bottom is shown in figure 9. By plotting the data logarithmically, it was found that the first part of the curve, as far as $x = 3m$, follows very closely the empirical relation

$$r = 1 - 0{\cdot}24x^{0{\cdot}615}, \qquad (1)$$

where x is in metres.

From the correlograms of figure 4, which have been continued until r_τ approaches the axis of τ, it is possible to compute the longitudinal scale L of the turbulence, according to the definition

$$L = U\int_0^\infty r_\tau \, d\tau.$$

For record 41 *a*, $L = 8{\cdot}0$ m, and for 43 *a*, $L = 6{\cdot}88$ m. These values are approximately half the depth of the water.

4·7. *Comparison with locally isotropic turbulence*

The empirical relation for r given in the preceding paragraph is of a form similar to the relation given by the theory of locally isotropic turbulence (see, for example, Batchelor 1947), which has been confirmed experimentally for the turbulence behind a grid in a wind-tunnel. In this case the relation is

$$f(x) = 1 - C'x^{\frac{2}{3}} \quad (2)$$

for $\eta \ll x \ll L$, where

$$C' = \frac{1}{2}\frac{C\epsilon^{\frac{2}{3}}}{[u^2]},$$

C being a constant, whose value is found empirically to be about 1·5,

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}} = \text{'local scale of turbulence'}, \quad (3)$$

$$L = \int_0^\infty f(x)\,\mathrm{d}x = \text{'integral scale of turbulence'}, \quad (4)$$

ϵ = rate of dissipation of turbulent energy per unit mass, and ν is the kinematic viscosity.

For isotropic turbulence,

$$\epsilon = \frac{15\nu[u^2]}{\lambda^2}, \quad \text{where} \quad \lambda = -f''(0). \quad (5)$$

The conditions in a strongly shearing current near the bottom seem, at first sight, to be so unfavourable to the occurrence of local isotropy in the turbulence that it would appear unwise to regard the similarity between equations (1) and (2) as indicating a degree of local isotropy in this case. A more reasonable interpretation might be to suggest that the initial form of the $f(x)$ curve is comparatively insensitive to deviations from local isotropy.

With this reservation, it appears instructive nevertheless to make estimates of the orders of magnitude of the quantities ϵ, η and λ, assuming that they may be regarded as having some significance in the turbulence of a tidal current. Such estimates have been made by Batchelor (1950) for turbulence in the lower layers of the atmosphere.

In the steady state, the rate of dissipation of turbulent energy by viscosity must equal the rate of doing work by the external forces on the mean motion. The dissipation of energy of the mean motion directly by viscosity may be neglected. Thus, if the mean current from surface to bottom is U, the frictional stress at the bottom is F, given by $F = k\rho U^2$ per cm^2, and the depth of water is h, the mean value of ϵ from surface to bottom is given by

$$\epsilon = kU^3/h\,\mathrm{cm}^2\,\mathrm{s}^{-3}.$$

If $k = 0{\cdot}002$, $U = 100\,\mathrm{cm/s}$, $h = 20\,\mathrm{m}$,

$$\epsilon = 1\,\mathrm{cm}^2\,\mathrm{s}^{-3} \text{ approximately.}$$

This value may be compared with Taylor's (1919) estimate of $0{\cdot}08\,\mathrm{cm}^2\,\mathrm{s}^{-3}$ for the mean rate of dissipation of tidal energy in the Irish Sea as a whole.

With $\epsilon = 1\,\mathrm{cm}^2\,\mathrm{s}^{-3}$ and taking $\nu = 10^{-2}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$, $\eta = 10^{-1{\cdot}5}\,\mathrm{cm} = 0{\cdot}03\,\mathrm{cm}$, approximately.

If $[u^2]$ is taken as $25\,\mathrm{cm}^2\,\mathrm{s}^{-2}$, equation (5) gives $\lambda = 2\,\mathrm{cm}$, approximately.

As shown in § 4·6, L was estimated at 6·88 and 8·0 m from two particular records.

Thus the orders of magnitude of the three characteristic lengths η, λ and L are probably such that η is less than 1 mm, λ is a few centimetres and L a few metres.

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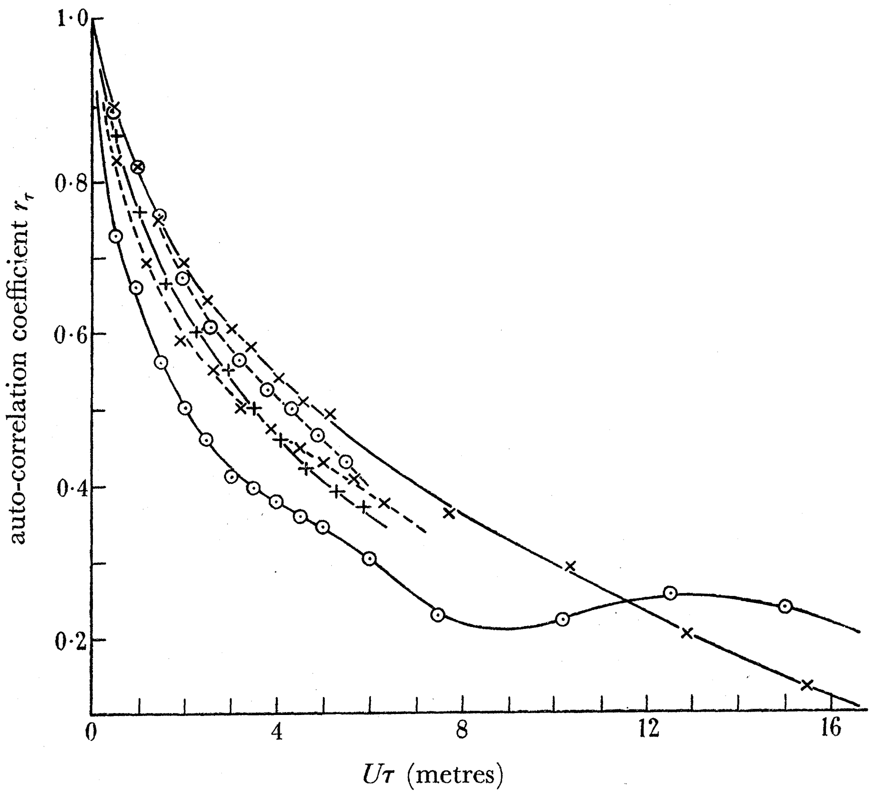

FIGURE 8. Curves of r_τ as a function of $U\tau$ for records near the bottom. —⊙— record 41*a*, --×-- record 41*f*, --⊙-- record 42*a*, —×— record 43*a*, —+— record 45*c*.

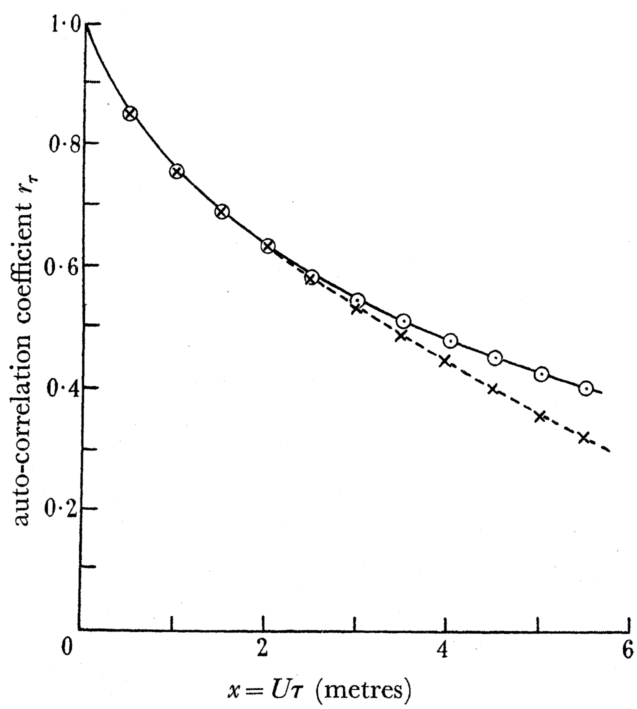

FIGURE 9. Mean curve of r_τ as a function of $x = U\tau$ for records near the bottom. —⊙— mean of observations, --×-- calculated from the equation $r = 1 - 0{\cdot}24x^{0{\cdot}615}$.

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4·8. *Comparison of longitudinal, lateral and vertical distance-correlations*

In table 4, column 17, are given values of the time interval τ_1 for which the auto-correlation coefficient r_τ is equal to r, the distance-correlation coefficient in either the vertical or lateral directions, depending on the record considered. Column 18 gives U the average of the mean currents recorded by the two meters and column 19 the product $U\tau_1$ which, on the assumption of § 4·6, represents the separation in the direction of flow corresponding to a distance-correlation coefficient r_τ. The value of $U\tau_1$ in column 19 may therefore be compared with the vertical separation in column 2 or the horizontal separation in column 3 to give an estimate of the ratio of the longitudinal scale of turbulence to the vertical or lateral scale respectively.

For records 10 *b*, 20 *b* and 32 *a*, taken above mid-depth, the values of r may be spuriously high, due to the effect of surface waves, as discussed in § 4·2. It seems inadvisable, therefore, to draw any conclusions from these cases.

Considering the records near the bottom, four records give the average ratio of $U\tau_1$ to the vertical separation as 2·9, and two records give the ratio of $U\tau_1$ to the horizontal separation as 3·9. If these results are valid, they may be taken as indicating that, within 2 m of the bottom, the lateral and vertical scales of turbulence are of the same orders of magnitude, while the scale in the direction of the mean current is of the order of three times as large.

The tentative nature of this conclusion should be stressed, however. A possible source of error would arise from random fluctuations at the individual meters superposed on the true turbulence which, as shown in § 4·3, would reduce the apparent distance-correlation coefficient. If random disturbances with a root-mean-square value 0·6 that of the turbulent fluctuations were present, the differences in the scales of turbulence reported above would be completely false, but it seems improbable that errors of this magnitude would have occurred.

5. Further investigations

The present series of observations has extended the previous results on the component of turbulent velocity in the direction of the mean flow and its auto-correlation with respect to time at a fixed point. It has also enabled a start to be made in the study of the lateral and vertical distance-correlations and scale of this component.

It appears that future progress should be sought in two main directions. (1) Further and more conclusive measurements of the distance-correlations, using instruments of dimensions small compared with the distance between them, so as to reduce interference with the flow to a minimum. (2) Simultaneous measurements of the vertical and lateral components of the turbulent velocity at a point, as well as the component in the direction of the mean flow. The Reynolds stresses could then be evaluated directly. It is hoped to develop experimental techniques which will enable such observations to be made.

It is a pleasure to record our thanks to Professor J. Proudman, F.R.S. to whom this investigation of turbulence is due, for his continued interest in the work. The experimental work was carried out jointly by the University Department of Oceanography and the Liverpool Observatory and Tidal Institute, and several members of the staff of each institution took part.

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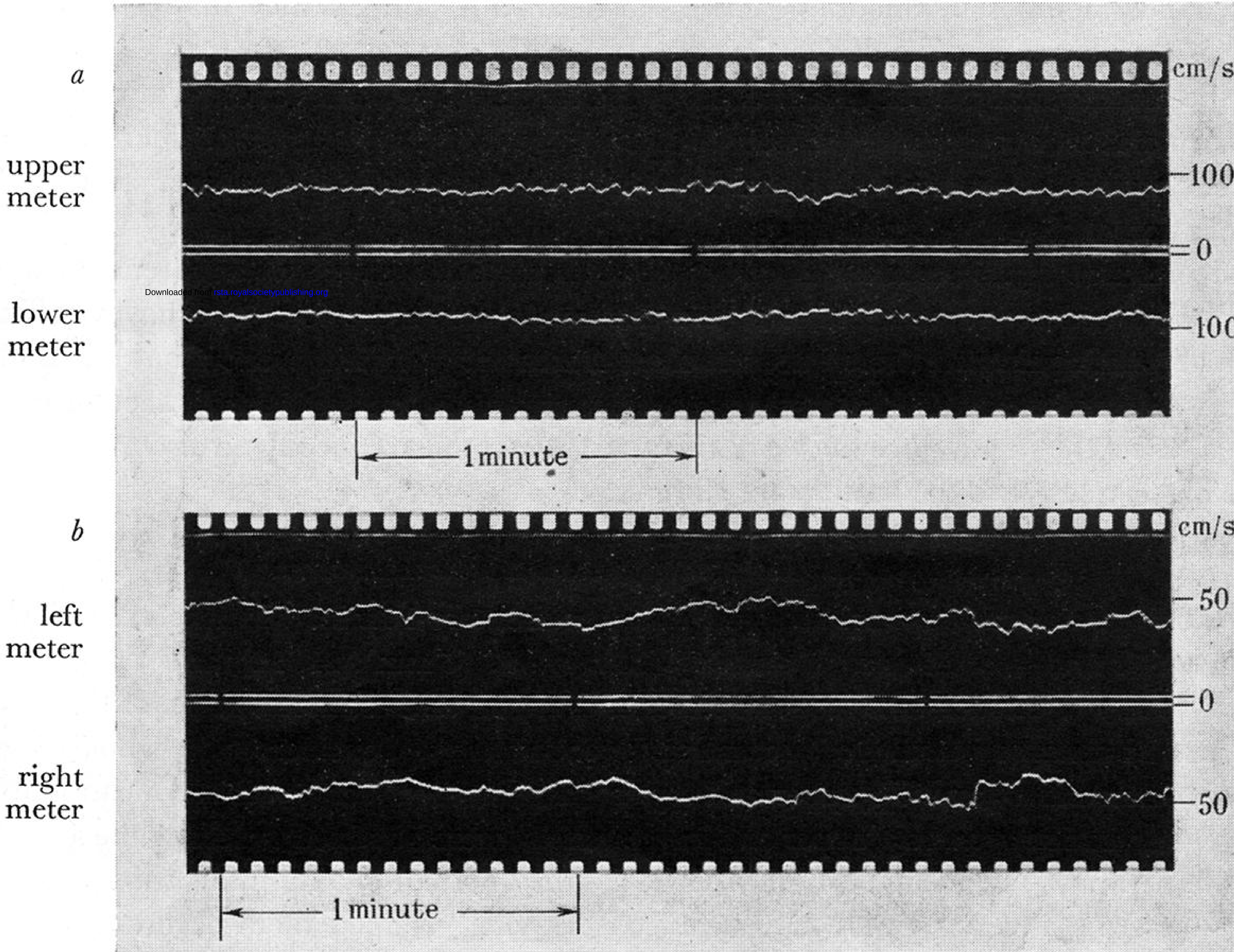

FIGURE 1. Two examples of simultaneous recordings by two current meters. (*a*) part of record 32*b*, meters 7·3 and 9·0 m below the surface; (*b*) part of record 43*c*, meters separated horizontally by 75 cm, height 1·2 m above the bottom.